Random Matrices and Applications (5th – 9th June 2023 in Kyoto, Japan) Program and Abstracts

Schedule

	5th (Mon)	6th (Tue)	7th (Wed)	8th (Thu)	9th (Fri)
9 : 00 – 9 : 20	Registration	Coffee	Coffee	Coffee	Coffee
9 : 20 – 9 : 30	Welcoming				
9 : 30 – 10 : 20	Bordenave	Yau	Menon	Hasebe	Schnelli
10 : 20 – 10 : 50	Coffee	Coffee	Coffee	Coffee	Coffee
10 : 50 – 11 : 40	Ghosh	Hayase	Yoshida	Basak	Matsumoto
11 : 40 – 12 : 30	Wang	McSwiggen	Banna	Lunch(11:40-13:00)	Kieburg
12 : 30 – 13 : 40	Lunch	Lunch	Lunch	Xu(13:00-13:50)	Lunch
13 : 40 – 14 : 30	Sato	Katori	Free discussion	Poster talk(13:50-)	Arizmendi
14 : 30 – 15 : 00	Coffee	Coffee	Free discussion	Poster	Rudelson
15 : 00 – 15 : 50	Seo	Sahasrabudhe	Free discussion	Poster	
15 : 50 – 16 : 40	Osada	Parraud	Free discussion	Nikeghbali	

Abstracts

5th (Mon)

- **Charles Bordenave (Aix-Marseille University)**

Title: *Norm of matrix-valued polynomials in random unitaries and permutations*

Abstract: This is joint work with Benoit Collins (Univ. Kyoto). We consider a non-commutative polynomial in several independent N-dimensional random unitary matrices, uniformly distributed over the unitary, orthogonal or symmetric groups, and assume that the coefficients are n-dimensional matrices. We study the operator norm of this random non-commutative polynomial. We compare it with its counterpart where the the random unitary matrices are replaced by the unitary generators of the free group von Neumann algebra. Our first result is that these two norms are overwhelmingly close to each other in the large N limit, and this estimate is uniform over all matrix coefficients as long as $n \leq \exp(N^{\alpha})$ for some explicit $\alpha > 0$. Our result provides a new proof of the Peterson-Thom conjecture. Our second result is a universal quantitative lower bound for the operator norm of polynomials in independent N-dimensional random unitary and permutation matrices with coefficients in an arbitrary C^*-algebra. A variant of this result for permutation matrices generalizes the Alon-Boppana lower bound in two directions.

- **Subro Ghosh (National University of Singapore)**

Title: *Gaussian fluctuations for spin systems and point processes: near-optimal rates via quantitative Marcinkiewicz's theorem*

Abstract: We investigate a very general technique to obtain CLTs with near-optimal rates of convergence for broad classes of strongly dependent stochastic systems, based on the zeros of the characteristic function. Using this, we demonstrate Gaussian fluctuations for the magnetization (i.e., the total spin) for a large class of ferromagnetic spin systems on Euclidean lattices, in particular those with continuous spins, at the near-optimal rate of $O(\log|\Lambda| \cdot |\Lambda| - 1/2)$ for system size $|\Lambda|$. This includes, in particular, the celebrated XY and Heisenberg models under ferromagnetic conditions. Our approach leverages the classical Lee-Yang theory for the zeros of partition functions, and subsumes as a special case a technique of Lebowitz, Ruelle, Pittel and Speer for deriving CLTs in discrete statistical mechanical models, for which we obtain sharper convergence rates. In a very different application, we obtain CLTs for linear statistics of a wide class of point processes known as α-determinantal processes which interpolate between negatively and positively associated random point fields (including the usual determinantal, permanental and Poisson processes). Notably, we address strongly correlated processes in dimensions ≥ 3, where connections to random matrix theory are not available, and handle a broad class of kernels including those with slow spatial decay (such as the Bessel kernel in general dimensions). A key ingredient of our approach is a broad, quantitative extension of the classical Marcinkiewicz Theorem that we establish under the significantly milder condition that the characteristic function is non-vanishing only on a bounded disk. Joint work with T.C. Dinh, H.S. Tran and M.H. Tran.

- **Ke Wang (Hong Kong University of Science and Technology)**

Title: *Random perturbation of low-rank matrices*

Abstract: The analysis of large matrices is a key aspect of high-dimensional data analysis, with computing the singular values and vectors of a matrix being a central task. However, real-world data is often disturbed by noise, which affects the essential spectral parameters of the matrix. While classical deterministic theorems can provide accurate estimates for the worst-case scenario, this talk will focus on the case when the perturbation is random. By assuming that the data matrix has a low rank, optimal subspace perturbation bounds can be achieved under mild assumptions. This talk is based on joint works with Sean O'Rourke and Van Vu.

- **Ryosuke Sato (Chuo University)**

Title: *Determinantal point processes and gauge-invariant CAR algebras*

Abstract: Determinantal point processes appear in various branches of mathematics and mathematical physics. In this talk, we will focus on determinantal point processes on discrete spaces, which arise from representation theory, among other things. First, I will explain how they are related to (quasi-free states of) gauge-invariant CAR algebras. After that, we will investigate a Markov process whose invariant measure is given by a determinantal point process. Using the

relationship between determinantal point processes and gauge-invariant CAR algebras, we will describe a generator of such a process in operator algebraic language.

- **Seo Seong-Mi (Chungnam National University)**

Title: *Partition functions of determinantal and Pfaffian Coulomb gases with radially symmetric potentials*

Abstract: In this talk, I will discuss two-dimensional Coulomb gas models which have determinantal or Pfaffian structures. I will present the asymptotic expansion of the partition functions when the underlying field is radially symmetric. We find that some correction terms in the asymptotic expansion contain topological information of the droplet, such as the Euler characteristic. I will explain how to obtain these correction terms. This is based on joint work with Sung-Soo Byun and Nam-Gyu Kang.

- **Hirofumi Osada (Chubu University)**

Title: *Geometric and dynamical rigidity of random point fields*

Abstract: Recently, various geometric rigidities have emerged for the Ginibre random point and the planar GAF. In this talk, we introduce a new type of rigidity called decomposability of random point fields, and, using this, we show the tagged particles of the Ginibre interacting Brownian motion are sub-diffusive.

6th (Tue)

- **Horng-Tzer Yau (Harvard University)**

Title: *Spectral statistics of random graphs*

Abstract: We will review the recent results regarding the edge statistics of Erdos-Renyi and random d-regular graphs. The main topics are Tracy-Widom law for the edge eigenvalues and Ramanujan graphs.

- **Tomohiro Hayase (Cluster Metaverse Lab)**

Title: *Asymptotic freeness in multilayer perceptron and related topics*

Abstract: This talk discusses the application of Free Probability Theory in dealing with mathematical challenges arising from random matrices in deep neural network (DNN) research. Specifically, we explore the asymptotic freeness of layerwise Jacobians in multilayer perceptron (MLP) models and related topics. The proof of asymptotic freeness relies on the invariance property of the MLP. We also treat Neural Network Gaussian Process and Neural Tangent Kernel, which are related to the information propagation during feedforward and backward propagation in deep neural networks. Furthermore, this talk presents novel insights into practical models beyond MLP, offering a more practical perspective.

- **Colin McSwiggen (NYU Courant)**

Title: *Dunkl Theory at Large N*

Abstract: Dunkl theory is an extremely versatile special functions theory that vastly generalizes the classical theory of hypergeometric and Bessel functions. The generalized Bessel functions of Dunkl theory include, as special cases, most of the spherical integrals whose large-N asymptotics have captured the attention of random matrix theorists for more than two decades. I will give a brief introduction to Dunkl theory and then present a result on the large-N limits of generalized Bessel functions, which unifies several results on spherical integrals in the random matrix theory literature. These limits follow from a large deviations principle for radial Dunkl processes, which are generalizations of Dyson Brownian motion. I will also discuss a large deviations principle for the even more general family of radial Heckman-Opdam processes, as well as an application to asymptotic representation theory. Joint work with Jiaoyang Huang.

- **Makoto KATORI (Chuo University)**

Title: *Eigenvalues, eigenvector-overlaps, and regularized Fuglede–Kadison determinant of the non-Hermitian matrix-valued Brownian motion*

Abstract: The non-Hermitian matrix-valued Brownian motion is the stochastic process of a random matrix whose entries are given by independent complex Brownian motions. The bi-orthogonality relation is imposed between the right and the left eigenvector processes, which allows for their scale transformations with an invariant eigenvalue process. The eigenvector-overlap process is a matrix-valued process, each element of which is given by a product of an overlap of right eigenvectors and that of left eigenvectors. We derive a set of stochastic differential equations for the coupled system of the eigenvalue process and the eigenvector-overlap process and prove the scale-transformation invariance of the system. The Fuglede–Kadison (FK) determinant associated with the present matrix-valued process is regularized by introducing an auxiliary complex variable. This variable is necessary to give the stochastic partial differential equations (SPDEs) for the time-dependent random field associated with the regularized FK determinant and for its logarithmic variation. Time-dependent point process of eigenvalues and its variation weighted by the diagonal elements of the eigenvector-overlap process are related to the derivatives of the logarithmic random-field of the regularized FK determinant. From the SPDEs a system of PDEs for the density functions of these two types of time-dependent point processes are obtained. The present talk is based on the joint work with Syota Esaki (Fukuoka) and Satoshi Yabuoku (Kitakyushu).

- **Julian Sahasrabudhe (University of Cambridge)**

Title: *The singularity probability of a random symmetric matrix*

Abstract: Let A be matrix drawn uniformly from all n by n symmetric matrices with entries in $\{-1, 1\}$. We consider the following basic problem: what is the probability that A is singular? While there has been quite a bit of success on the "iid" version of this problem, a good understanding of the problem for symmetric matrices has remained elusive.

In this talk I will discuss the proof that this singularity probability for symmetric matrices is exponentially small; which is sharp up to the constant in the base of the exponent.

This is based on joint work with Marcelo Campos, Matthew Jenssen and Marcus Michelen.

- **Felix Parraud (KTH Royal Institute of Technology)**

Title: *Asymptotic expansions in Random Matrix Theory and application: the case of Haar unitary matrices*

Abstract: Recently we developed a method to compute asymptotic expansions of certain quantities coming from Random Matrix Theory. More precisely if one considers the expectation of the trace of a sufficiently smooth function evaluated in a random matrix, one can compute a Taylor expansion (in the dimension of our random matrix) of this quantity. This method relies notably on free stochastic calculus. In a previous work we studied polynomials in independent GUE random matrices, in this talk we consider the case of Haar unitary matrices. We shall explain the additional difficulties that this model brings and give a few applications of this result to Random Matrix Theory as well as links with Weingarten calculus.

7th (Wed)

- **Govind Menon (Brown University)**

Title: *Stochastic Nash evolution*

Abstract: We present a probabilistic formulation of the embedding problem for Riemannian manifolds. The main goal is to construct Gibbs measures supported on isometric embeddings. The main results are geometric stochastic PDE with the structure of a Riemannian Langevin equation. This structure will be introduced using matrix models of independent interest. This framework emerged through a study of the Nash embedding theorems in light of h-principles for PDE developed by De Lellis and Szekelyhidi. Their work used the Nash embedding theorems as a tool for the Euler equations and turbulence. We use this analogy in the other direction, using ideas from statistical physics to study embedding. This gives this "bridge" new depth, providing insights into optimization and geometry, in addition to turbulence. In particular, we will outline a path from universality in RMT to universality in turbulence and geometry. This is joint work with Dominik Inauen (University of Leipzig).

- **Hiroaki YOSHIDA (Ochanomizu University, Tokyo)**

Title: *An application of fluctuation moments of random matrices to statistical data analysis*

Abstract: This talk contains the results of the joint work with Ayako Hasegawa (OU) and Noriyoshi Sakuma (NCU). For a sequence of random matrices $\big(\boldsymbol{X}_N\big)_{N\in\mathbb{N}}$, the moments are given by the first order limits

$$\alpha_k = \lim_{N\to\infty} \mathrm{E}\big[\mathrm{tr}\big(\boldsymbol{X}_N^k\big)\big],$$

where tr denotes the normalized trace. Then the second order limits

$$\alpha_{p,q} = \lim_{N\to\infty} \mathrm{Cov}\big(\mathrm{Tr}\big(\boldsymbol{X}_N^p\big), \mathrm{Tr}\big(\boldsymbol{X}_N^q\big)\big) = \lim_{N\to\infty} \mathrm{E}\big[\mathrm{Tr}\big(\boldsymbol{X}_N^p - \alpha_p \mathbf{1}\big)\cdot \mathrm{Tr}\big(\boldsymbol{X}_N^q - \alpha_q \mathbf{1}\big)\big]$$

where Tr denotes the unnormalized trace, that is, the simple sum of the diagonal entries, are called the fluctuation moments.

First, we see the formula of the fluctuation moments for the compound Wishart matrices. It can be found that the sample covariance matrix with dependent entries of MA modeled Gaussian processes can be viewed as the compound Wishart matrix, which implies that the limit spectral measure in Marchenko-Pastur limit with the asymptotic ratio λ is given as the compound free Poisson law of parameter λ with the compounded measure is derived from the autocovariance function of the MA model for the dependence.

Applying the formula of the fluctuation moments to the above compound Wishart matrices, we propose the statistical hypothesis testing of the time series model, that is, a test for the goodness of the fitting to the given model, which is one of applications of fluctuation moments.

If we have a time we also give another application of fluctuation moments to the regularization problem in non-linear kernel regression, which is ensured by the fact that Gaussian kernel random matrices can be regarded as Wihsart matrices.

- **Marwa Banna (NYU Abu Dhabi)**

Title: *Quantitative estimates on random matrices using free probability tools*

Abstract: In this talk, we highlight first the connection between free probability theory and random matrices. We then show how free probability tools can be used to obtain regularity properties of limiting spectral distributions and how to quantify such convergences for some random matrix models. In particular, we consider a class of random block matrices and derive quantitative estimates on the level of Cauchy transforms that can be passed to the Kolmogorov distance in some cases. The content of the talk is based on results from joint works with G. Cébron and T. Mai.

8th (Thu)

- **Takahiro Hasebe (Hokkaido University)**

Title: *Principal submatrices of unitarily invariant hermitian random matrices*

Abstract: The empirical eigenvalue distribution of a principal submatrix of rotatially invariant random matrices is close to that of the original matrix in the large size limit. The difference of them with a suitable scaling can be described by the Markov-Krein transform, which was known as a folklore theorem. In this talk a free probabilistic proof is given. The main machinery is the Weingarten calculus and noncrossing partitions. If possible, a recent progress will also me mentioned. This is a joint work with Katsunori Fujie.

- **Anirban Basak (ICTS)**

Title: *Large Deviations of the largest eigenvalue of supercritical sparse Wigner matrices*

Abstract: Let X be an $n \times n$ sparse random symmetric matrix that is a Hadamard product of G and Ξ, where G is a Wigner matrix with centered sub-Gaussian entries (the distribution of the diagonals is allowed to be different) and Ξ is a symmetric matrix with i.i.d. Bernoulli p entries on and above the diagonal positions. In this talk, we will discuss the large deviations of the largest eigenvalue of $X/\sqrt{np}$ under the assumption that $\log n/n \ll p \ll 1$. As a byproduct, we also obtain the large deviations of the second largest eigenvalue of the adjacency matrix of a sparse Erdős-Rényi graph. This is based on joint work with Fanny Augeri.

- **Yuanyuan Xu (Institute of Science and Technology Austria)**

Title: *Universality for the rightmost eigenvalue of non-Hermitian random matrices*

Abstract: I will report some recent progress on the universality for the extreme eigenvalue of a large random matrix with i.i.d. entries. Beyond the radius of the celebrated circular law, we will establish a precise three-term asymptotic expansion for the rightmost eigenvalue with an optimal estimate on the error term. Based on this result, we will further show that the properly normalized rightmost eigenvalue will converge to a Gumbel distribution, as the dimension goes to infinity. Similar results will also hold true for the spectral radius of the matrix.

- **Ashkan Nikeghbali (University of Zürich)**

Title: *Random matrices and (stochastic) zeta functions*

Abstract: In this talk we describe two different ways of obtaining some remarkable random holomorphic functions as scaling limit of characteristic polynomials of different random matrix ensembles (or more generally from a converging sequence of point processes). We then describe a few more remarkable limit theorems in this setting and connect it to some (conjectural) behaviour of the Riemann zeta function. We also discuss some universal aspects of these limiting random functions.

9th (Fri)

- **Kevin Schnelli (KTH Royal Institute of Technology)**

Title: *Quantitative Tracy-Widom law for generalized Wigner random matrices*

Abstract: We will discuss a quantitative Tracy-Widom law for the largest eigenvalue of generalized Wigner random matrices. More precisely, we will prove that the fluctuations of the largest eigenvalue of a generalized Wigner matrix of size N converge to its Tracy-Widom limit at a rate nearly N-1/3, as N tends to infinity. Our result follows from a quantitative Green function comparison theorem, originally introduced by Erdos, Yau and Yin to prove the edge universality, on a finer spectral parameter scale with improved error estimates and without second moment matching. The proof relies on the continuous Green function interpolation with the Gaussian invariant ensembles. Precise estimates on leading contributions from the second, third and fourth order moments of the matrix entries are obtained using iterative cumulant expansions along with uniform convergence estimates for correlation kernels of the Gaussian invariant ensembles. This is joint work with Yuanyuan Xu (IST Austria).

- **Sho Matsumoto (Kagoshima University)**

Title: *Law of large numbers for Schur–Weyl measures on shifted Young diagrams*

Abstract: The Plancherel measure of the symmetric group gives a basic random Young diagram. The limit shape of a Plancherel Young diagram is well known as the Logan–Shepp–Vershik–Kerov curve. In this talk, we consider probability measures on shifted Young diagrams which appear in spin representations of symmetric groups. Spin representations acting on tensor products provide one such measure, the shifted Schur–Weyl measure. We characterize the corresponding limit shape by free cumulants associated with Young diagrams. This talk is based on joint work with Piotr Śniady (Sel. Math. 2020).

- **Mario Kieburg (University of Melbourne)**

Title: *Computing the Typical Entanglement Entropy with Random Matrices*

Abstract: The information theoretical properties of a generic quantum state have been of interest longer than Page's work in the 90's which has been about the typical entanglement entropy of a generic pure state. The idea has been to use the geometric properties of the manifold of pure states to define a uniform or natural probability measure for pure quantum states. For arbitrary pure states in Hilbert space this is the uniform measure on a sphere which has been Page's setting. Then, the corresponding reduced density operator is a random matrix. The situation becomes more involved when considering sub-manifolds such as Gaussian pure states as those are not described by spheres but by more complicated cosets. The corresponding density matrices are embedded random matrices, meaning matrices which are very sparse, and their covariance matrices are related to classical random matrix ensembles. Recently, we could derive closed expressions for the entanglement entropy with the help of random matrix theory. I will review these recent results of bipartite systems and current progress in tripartite systems. These are joint projects with Eugenio Bianchi, Lucal Hackl, Marcos Rigol and Lev Vidmar.

- **Octavio Arizmendi (CIMAT)**

Title: *Universality of free random variables: atoms for non-commutative rational functions*

Abstract: Consider a tuple $(Y_1, ..., Y_d)$ of normal operators in a tracial operator algebra setting with prescribed sizes of the eigenspaces for each Y_i. We address the question: what can one say about the sizes of the eigenspaces for any non-commutative polynomial $P(Y_1, ..., Y_d)$ in those operators? We show that for each polynomial P there are unavoidable eigenspaces, which occur in $P(Y_1, \ldots, Y_d)$ for any $(Y_1, ..., Y_d)$ with the prescribed eigenspaces for the marginals. We further show that this minimal situation is given when Y_i's are freely independent random variables with prescribed atoms in their distributions. Finally, we give applications of this result in free probability and in spectral theory of graphs.

- **Mark Rudelson (University of Michigan)**

Title: *Approximately Hadamard matrices and random frames*

Abstract: An n by n matrix with plus-minus 1 entries which acts as a scaled isometry is called Hadamard. Such matrices exist in some, but not all dimensions. Combining number-theoretic and probabilistic tools we construct matrices with plus-minus 1 entries which act as scaled approximate isometries for any n. More precisely, the matrices we construct have condition numbers bounded by a constant independent of the dimension. We will also discuss an application in signal processing. A frame is an overcomplete set of vectors which allows a robust decomposition of any vector in the space as a linear combination of these vectors. Frames are used in signal processing since the loss of a fraction of coordinates does not prevent the recovery of the signal. We will discuss a question when a random frame contains a copy of a nice basis.

Joint work with Xiaoyu Dong.

Poster talk

- **Nicolas Delporte (OIST)**

Title: *Eigenvalues of Random Tensors with Field Theoretic Methods*

Abstract: We will present recent results on the distribution of real eigenvalues for symmetric Gaussian random tensors, using supersymmetric representations and large-N Dyson-Schwinger equations, as well as hints to 1/N corrections. Further, we will show a phase transition when a Gaussian noise is added to the eigenvalue equation.

- **PIERFRANCESCO DIONIGI (Leiden University)**

Title: *Spectral Breaking of Ensemble Equivalence*

Abstract: In this poster we will revise the concept of Breaking of Ensemble Equivalence (BEE) and its connections with large deviations theory and random matrix theory. We will see how spectral properties of the adjacency matrix of certain random graph ensembles can be used to detect BEE. In particular we are interested in the connections between BEE and the largest eigenvalue of these ensembles in the BBP phase transition regime. This is a poster from a series of ongoing works together with Diego Garlaschelli, Rajat S. Hazra, Frank den Hollander and Michel Mandjes.

- **Trinh Khanh Duy (Waseda University)**

Title: *A new model of associated Jacobi polynomials related to random matrix theory*

Abstract: A family of associated Jacobi polynomials was introduced in the work of Wimp (1987) by shifting coefficients in the three-term recurrence relation of Jacobi polynomials by a positive number. Ismail and Masson (1991) then studied a slightly different model in relation with birth and dead processes. A new model of associated Jacobi polynomials has been introduced recently by Trinh and Trinh (2021) when considering Jacobi beta ensembles in a high temperature regime. In this presentation, we show some properties of the new model arisen in the study of Jacobi beta ensembles. This is based on a joint work with Fumihiko Nakano (Tohoku University) and Hoang Dung Trinh (VNU University of Science, Hanoi).

- **Syota Esaki (Fukuoka University)**

Title: *Stochastic Differential Equations of eigenvalues, eigenvectors and overlaps of non-Hermitian matrix-valued Brownian motion*

Abstract: In this presentation, we report stochastic processes associated with eigenvalues and eigenvectors of the non-Hermitian matrix-valued Brownian motion.

The non-Hermitian matrix-valued Brownian motion(nHBM) is a dynamical $N \times N$ random matrix model whose entries are given by i.i.d. complex Brownian motions.

The eigenvalue process associated with nHBM are related to the Ginibre ensemble. As matrix-valued Brownian motion, for example, the eigenvalue process for the Hermitian matrix-valued Brownian motion, which assumes the Hermitian symmetry, has been well studied. This process is known to be the solution of the SDE of the Dyson Brownian motion for $\beta = 2$. On the other hand, the eigenvalue process of nHBM is difficult to give the SDE using only themselves, and the SDE is given by combining the eigenvalue process and the eigenvector process. However, in general, eigenvectors are not uniquely determined from the matrix. Hence, it is necessary to pay attention to how the eigenvectors are determined when considering the time evolution of the eigenvectors. In this presentation, we consider the time evolution of the overlaps associated with nHBM, and give an expression formula for the time evolution. Overlaps are quantities that express the non-normality of the matrix, and they appear in the quadratic variations of the eigenvalue process. Hence, it is important for the analysis of the eigenvalue and eigenvector processes. In addition, we can see the representation formula does not depend on how the eigenvectors are taken. This is a joint work with Makoto KATORI (Chuo University) and Satoshi YABUOKU(National Institute of Technology, Kitakyushu College).

- **Katsunori Fujie (Hokkaido University)**

Title: *Combinatorial approach to limit theorems in finite free probability*

Abstract: Since the 2010s, when Marcus, Spielman, and Srivastava solved the Kadison–Singer conjecture and found a connection between its solution and free probability theory, this research area has been called finite free probability. Much progress has been made recently, and of particular interest are finite free cumulants by Octavio and Perales, where free cumulants are the basic tool used as a discretization for the characteristic function in the context of free probability. Just recently, the speaker, Octavio Arizmendi (CIMAT) and Yuki Ueda (Hokkaido Education University) have proved a few limit theorems in finite free probability by a unified approach using finite free cumulants in arXiv:2303.01790. The purpose of this poster is to introduce our approach.

- **Hao-Wei Huang (National Tsing Hua University)**

Title: *Additivity Violations of Random Quantum Channels of non-white Wishart Types*

Abstract: Most core problems in quantum information theory have elementary formulations but still resist solutions, one of which is the additivity conjecture of the minimum output entropy of quantum channels. All previously known results, including extensive numerical work, are consistent with the conjecture until it was shown to be false by Hasting and successive works by others. In this talk, we will briefly introduce the history and developments regarding this problem, present our random quantum channels composed of non-white Wishart ensembles and explore their additivity violations. The noted additivity violations occurring in our constructed random quantum channels are acquired by utilizing random matrix theory.

- **Kiran Kumar (Indian Institute of Technology, Bombay)**

Title: *A unified approach for fluctuations of linear eigenvalue statistics of patterned random matrices*

Abstract: A patterned random matrix is defined as $A_n = \big(x_{L(i,j)}\big)$, where the input sequence $\{x_i\}$ is a sequence of independent random variables with mean zero, variance one and uniformly bounded higher moments, and L is known as the link function. Several important classes of matrices, including the Wigner Matrix, Toeplitz matrix, Hankel matrix, circulant-type matrices and the block versions of these matrices, fall into this category. In 2008, Bose and Sen showed that under some restrictions on the link function L, the corresponding patterned matrices always have a sub-Gaussian limiting spectral distribution. In this work, we study the linear eigenvalue statistics of $A_n/\sqrt{n}$, given by

$$\eta_p = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \lambda_i^p$$

where λ_i are the eigenvalues of $A_n/\sqrt{n}$. We show that when p is even and under similar restrictions on $L, \eta_{\mathrm{p}} - \mathbb{E}\eta_p$ converges in distribution either to a normal distribution or to a degenerate distribution as $n \to \infty$. We show that under further assumptions on L, the limit is always a normal distribution. We show that for Toeplitz, Hankel, circulant-type matrices and block versions of these matrices the linear eigenvalue statistics for even degree monomials converge to a normal distribution with mean zero.

We also derive the limiting moments for η_p when p is odd and show that the linear eigenvalue statistics may not converge to a normal distribution. We also study the linear eigenvalue statistics of patterned matrices with independent Brownian motion entries. We show that under the assumption that if a non-degenerate limit exist for the linear eigenvalue statistics of a patterned random matrix with independent entries, then the linear eigenvalue statistics with Brownian motion entries converge to a deterministic real-valued process.

- **Kohei Noda (Kyusyu University)**

Title: *Determinantal structure of conditional overlaps for the induced Ginibre unitary ensemble*

Abstract: As in widely known, a non-Hermitian matrix features distinct left and right eigenvectors, which form a bi-orthogonal system. In this poster, we show the determinantal structure and its scaling limits for the conditional overlaps for the induced Ginibre unitary ensemble depending on the spectral parameters. As a consequence, we will confirm the universality of the overlaps and find new phenomenon for the overlaps. References

[1] G. Akemann, R. Tribe, A. Tsareas, and O. Zaboronski, On the determinantal structure of conditional overlaps for the complex Ginibre ensemble, Random Matrices: Theory and Applications 9 (2020), no. 04, 2050015. MR4133071

[2] G. Akemann, Y. Förster and M. Kieburg.: Universal eigenvector correlations in quaternionic Ginibre ensembles, J. Phys. A., 53, (2020), 145201.

[3] K. Noda: Determinantal structure of the overlaps of the induced spherical unitary ensemble, in preparation.

[4] K. Noda: Determinantal structure of the overlaps of the induced Ginibre unitary ensemble, in preparation.

- **Syota Osada (Kagoshima University)**

Title: *Logarithmic derivatives and dynamics of point processes on* $\mathbb{R}^d$

Abstract: The interest of this presentation is the construction of the dynamics of infinite particle systems using Dirichlet forms associated with point processes on $\mathbb{R}^d$. The construction of dynamics has been done via the evaluation of local density functions of point processes, such as the Dobrushin-Lanford-Ruelle equation and the quasi-Gibbs property. In this presentation, we show that the integrability of the logarithmic derivative, which is the derivative of the Campbell measure associated with a point process in the sense of distribution, implies the closability of the form.

- **Daniel Perales Anaya (Texas AM University)**

Title: *New results in Finite free probability and its relation to Random Matrices and Free Probability*

Abstract: We will introduce the finite free additive and multiplicative convolutions of two polynomials and explain how it is related random matrices using the characteristic polynomials, and how it relates to free probability in the limit. I will explain how finite free probability is used to study the effect of differentiating a sequence of polynomials several times and then looking at the resulting limiting root distribution (joint work with Octavio Arizmendi and Jorge Garza-Vargas). I we will discuss some new analogies between the finite free and the free world that are part of an ongoing project with Andrei Martinez-Finkelshtein and Rafael Morales.

- **Raunak Shevade (Indian Institute of Technology, Bombay)**

Title: *Test of independence based on large Kendall's correlation matrix*

Abstract: The test of independence based on Kendall's correlation matrix has been studied in the literature when the dimension grows at the same rate as the sample size. No analogous test has been explored in the literature when the dimension grows either slower or faster than the sample size. Here we focus on the latter two high-dimensional regimes. We establish the asymptotic normality of traces of finite degree polynomials in Kendall's correlation matrix. We then use these results to propose an asymptotic test of independence within components of the observations. Simulation studies show that the proposed test performs well in various situations.

- **Yuya Tanaka, Ayana Ezoe, Saori Morimoto and Makoto Katori (Chuo University)**

Title: *PDE of the potential field for eigenvalues and eigenvector-overlaps of the non-Hermitian matrix-valued BM*

Abstract: The non-Hermitian matrix-valued Brownian motion (BM) induces a coupled system of the eigenvalue process and the eigenvector-overlap process. Here the eigenvector-overlap process is a matrix-valued process, each element of which is given by a product of an overlap of right eigenvectors and that of left eigenvectors. We consider the time-dependent random field defined on the two-dimensional complex space using the notion of the reguralized Fuglede–Kadison determinant of the matrix-valued stochastic process. As explained in a talk by one of the present authors (MK), this random field plays an important role for the system. In this presentation, we take average of this random field to define a deterministic field and determine the partial differential equation (PDE) which this field satisfies. We call this field the potential field, since its derivatives give the time-dependent density functions of the eigenvalue process and the eigenvector-overlap process. The present consideration will provide a mathematical interpretation of the calculus reported in physics literature (e.g., Burda et al. (2015)). The exact solutions of the PDE and explicit expressions for the density functions are shown for several initial conditions. Comparison with numerical simulations will be also reported.

- **Sarah Timhadjelt (Aix-Marseille University)**

Title: *Spectral gap of convex combination of a random permutation and a bistochastic matrix*

Abstract: We consider a random bistochastic matrix of size N of the form $(1-r)M + rQ$ where $0 < r < 1, M$ is a uniformly distributed permutation and Q is a given bistochastic matrix. Under sparsity and regularity assumptions on the distribution of Q, we prove that the second largest eigenvalue $(1-r)M + rQ$ is essentially bounded by the spectral radius of an asymptotic equivalent independent of M.

- **Yuki Ueda (Hokkaido University of Education Asahikawa)**

Title: *Asymptotic behaviors of the free Lévy measure of the normal distribution*

Abstract: In 1980s, free probability theory was initiated by Voiculescu to attack some operator algebraic problem. The role of the normal distribution in free probability is played by Wigner's semicircle law from the free central limit theorem. On the other hands, a role of the classical

normal distribution in free probability is not very obvious and there are still some attempts to understand it. One of the most important results is free infinite divisibility for normal distribution by Belinschi, Bożejko, Lehner and Speicher (2011). Moreover, Hasebe, Sakuma and Thorbjørnsen (2019) proved that the normal distribution is freely selfdecomposable, that is, its free Lévy measure is of the form $k(x)/|x|dx$ for some nonnegative function k on $\mathbb{R}\setminus\{0\}$ which is nondecreasing on $(-\infty, 0)$ and nonincreasing on $(0, \infty)$. In this talk, we investigate a certain monotonicity, real analyticity and asymptotic behavior of the density of the free Lévy measure on the normal distribution. These results strengthen the known results for the normal distribution. This is a jointwork with Takahiro Hasebe.

- **Satoshi Yabuoku (National Institute of Technology, Kitakyushu College)**

Title: *Time evolution model of Gaussian beta ensemble and its eigenvalue processes*

Abstract: We consider symmetric tridiagonal matrix-valued processes and their eigenvalue processes. This matrix-valued process is defined by putting independent Brownian motions and Bessel processes on the diagonal entries and upper (lower)-diagonal ones, respectively. This model includes the time evolution model of Gaussian beta ensemble (GβE), which is the generalization of G(O/U/S)E in random matrix theory. In this presentation, we derive the SDEs which the eigenvalue processes satisfy, and we show that their minor eigenvalues appear in the SDEs. We also give the sufficient condition that the eigenvalue processes never collide with each other, by the dimensions of Bessel processes. References [1] S. Yabuoku, Eigenvalue processes of symmetric tridiagonal matrix-valued processes associated with Gaussian beta ensemble, arXiv:2204.00736